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GRAPHICAL SOLUTION OF FAULT PROBLEMS

BY
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PREFACE

The methods that the geological engineer uses in the study of ore deposits, have as yet received no satisfactory exposition in print, because of both the recentness of the development of this branch of engineering, and the extent of the subject, involving, as it does, the application of the methods of field geology, petrology, mineralogy, and the comparative study of ore deposits. It is not to be expected that the miner or engineer directing underground operations can always be thoroughly versed in the broad science of ore deposits, but he must at least make a study of the details and peculiarities of the deposit he is developing. He should understand, not only the mapping of underground workings, but of the structural features of the lode as well. If the structure is accurately plotted, the relation of the ore deposit to the same should become plain; the application of the methods of descriptive geometry becomes easy; and the developments may be carried on, not under the guidance of the miner's 'nose for ore,' but in accordance with the structure of the deposit. Thus a permanent record of the geology is kept that may mean much when portions of the mine become inaccessible.

The following pages present a revision of a series of articles appearing in the *Mining and Scientific Press* in June, July, and August, 1911. The fault planes and veins are represented ideally as true planes, the actual underground example which might have been used to illustrate the type cases, presenting irregularities and complications that unsuit them for use. Once the principles of graphic investigation are understood, little difficulty will be found in applying them to the mine maps. No originality is claimed in the method of projection used, on two (or more) horizontal planes, the interval between which is known. It has long been used in descriptive geometry and graphics, as well as to present certain fault relations in several books

on mining and geology. The scope of the book limits consideration to elementary type problems, and if the discussion of these contain anything new to the geological reader, this should emphasize that we may expect additions to our knowledge of the complex phenomena of faulting, from the application of accurate mathematical methods of investigation.

C. F. TOLMAN, JR.

Tucson, Arizona, September 19, 1911.

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GRAPHICAL SOLUTION OF FAULT PROBLEMS

I. INTRODUCTION.

A common occurrence in mining is to have a drift on the vein meet a polished wall, on the other side of which no ore is discovered. The life of the mine depends upon rediscovering the ore, but in many cases the vein is never found again. Occasionally the break is older than the ore deposit, and the ore solutions may not have passed through the fault gouge, and so no ore exists beyond the break; but in every case where the break is younger than the ore, the latter exists somewhere beyond the 'trouble,' as the Cornish miners call the fault. One superintendent may be unfortunate in his search; the next may have more knowledge or better luck, and when the vein is found he has scored a hit with the owners. Finally, if the ore is valuable and the superintendent unsuccessful, an expert in faulting may be called in to collect, study, and plot all data, but meanwhile important evidence may have been obliterated, which could have been observed when the fault was first found. For these reasons every man engaged in mining should be interested in fault problems.

The study of faults must be carried on according to the methods of applied geology, and the methods of detailed and exact geological mining investigation are of but recent development. For instance, in the somewhat generalized mapping of a large area, geologists once believed it only necessary to consider the vertical component of fault movement, an assumption which is entirely inadequate when accurate graphic investigations are undertaken. J. E. Spurr, in his excellent text-book 'Geology as Applied to Mining', emphasizes the occurrence of fault movement in all directions on the fault plane, but vertical sections, so universally used to portray generalized geological structure, show only the vertical component, and the importance of horizontal displacement, and pivotal fault motion, was

not thoroughly appreciated, until the San Francisco earthquake gave emphasis to these factors.

It is strange that in the standard text-books on geology, faulting has received such incomplete and unsatisfactory treatment, and so little attention has yet been paid to graphic methods of solution, especially when we remember that nearly thirty years ago R. W. Raymond published a summary of Hoefer's work on faulting,[1] involving the application of descriptive geometry to fault problems. Hoefer appreciated that faulting may involve movement in any direction on a fault plane, and that translatory movement may be combined with rotatory movement. With the exception of this article, no exposition of the graphic methods applicable to these problems has appeared in English until recently.[2] The engineer confronted by such problems must discover original solutions, remembering what he can of his early studies in 'Church's Descriptive Geometry,' or he probably overlooks the possibility and value of the graphic solution, and may not know just what data it is necessary to collect in order to solve the problem.

The problems presented below are selected from those given in the laboratory courses in structural geology at the School of Mines of the University of Arizona since 1905, by myself and assistants, especially Theodore B. Chapin. The methods given are the most simple of the many that I have used both in engineering practice and teaching. The discussion is confined as closely as possible to the graphic

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[1]Raymond, R. W. 'Hoefer's Methods of Determining Faults in Mineral Veins'. *Trans.* Amer. Inst. Min. Eng., Vol. 10, pp. 456-465.

[2]Reid, H. F. 'Geometry of Faults'. Bull. Geol. Soc. of America, Vol. 20, pp. 171-196. This valuable contribution goes over some of the ground covered in this discussion. The author does not adopt the simple concept presented by me in a following article regarding rotary fault movement. Mr. Reid conceives that poles of rotation may lie at various angles to the fault plane, and introduces artificial solutions, involving such movements as a rotation about, and translation along, one and the same pole.

solution of a set of type problems, and only the briefest possible statement of the various kinds of fault motion is made, with such a summary of fault nomenclature as is necessary for understanding them, and with a brief statement of the practical bearing of some of the solutions. The reader desiring a summary of the phenomena by which faulting is detected, as well as the modern ideas on the general subject of faulting, is referred to a paper entitled 'Methods of Investigating Problems in Faulting',[3] and to the valuable discussion in *Economic Geology*[4] for 1907, 'How Should Faults Be Named and Classified?'

II. NOMENCLATURE OF FAULT MOVEMENT

A direct measurement of the amount and direction of fault movement on the fault plane is, unfortunately, not often possible; at times only the direction of movement, or the perpendicular distance on the fault plane between the two broken portions of the faulted body, is known; or again, possibly only the components of the fault movement in a vertical plane, or in a horizontal plane, or in a plane perpendicular to the trace of the faulted body on the fault plane. Since measurements in all these different planes are made, and, further, since the complete set of measurements in any one plane often cannot be taken, an orderly set of names for all these is desirable, and is of great assistance in the discussion and solution of fault problems. Therefore the nomenclature previously suggested[5] is introduced here, with the substitution of the word 'normal' for the word 'vertical' throughout, on account of the criticism that the measurement there called 'vertical displacement' is not vertical, when the fault plane is inclined, but normal to a horizontal line. When rotation and translation are combined, these names are applicable

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[3]Tolman, C. F. *Mining Magazine*, New York, February 1906.

[4]*Economic Geology*, Vol. 1, No. 8; Vol. 2, No. 1, 2, 3, 4, 5, 6, and 7.

[5]Tolman, C. F. 'How Should Faults Be Named and Classified?' *Economic Geology*, Vol. 2, pp. 506-511.

to the movement at any given point of the break, the amount of the movement changing with the distance from the pole. In this case, however, a complete solution of the problem involves only the location of the pole about which the known angular rotation will produce the displacements observed; for it is evident that a combined rotation and translation can be considered as a pure rotation about an equivalent pole. The term 'angular displacement' may be used, and the location of the pole designated.

Measurements of fault movement are made in the following planes:

1. On the fault plane. All such measurements are designated as 'displacements'.
2. On a plane normal to the trace of the faulted body on the fault plane. These measurements are the 'separations'.
3. On any perpendicular plane. 'The throws'.
4. On a horizontal plane. (Generally the earth's surface.) 'The heaves'.

These are represented in Fig. 1 to 4 and in each diagram the vein or stratum is represented to contain a lens or nodule which has been split open by the fault movement, thus indicating at a glance the direction and amount of fault displacement.

The displacements. Fig. 1. The 'total displacement' is *ac*. This is taken as the hypothenuse of a triangle, *ab* being parallel and *bc* perpendicular to the trace of the faulted stratum on the fault plane; *ab* is called the 'parallel displacement', and *cb* the 'perpendicular displacement', and the angle $cab = \Theta$. With *ac* again as the hypothenuse, a second triangle is constructed with *cd* a horizontal line, and *ad* normal thereto; *ad* is called the 'normal displacement' and *cd* the 'horizontal displacement'.

The separations. Fig. 2. These, as defined above, are in a plane normal to the line *af*, and as this line happens to be horizontal, the measurements in this particular case are in a vertical plane, and are therefore both separations and throws. The 'total separation' *ac* lies in the fault plane

and has already been named 'perpendicular displacement'; *ad* is the 'parallel separation' and *cd* is the 'perpendicular separation', the latter being the only important measurement of this set. It is, of course, the shortest distance between the two faulted portions of the stratum or the faulted portions produced; *ce* and *ae* are, respectively, the 'horizontal' and 'normal separations'.

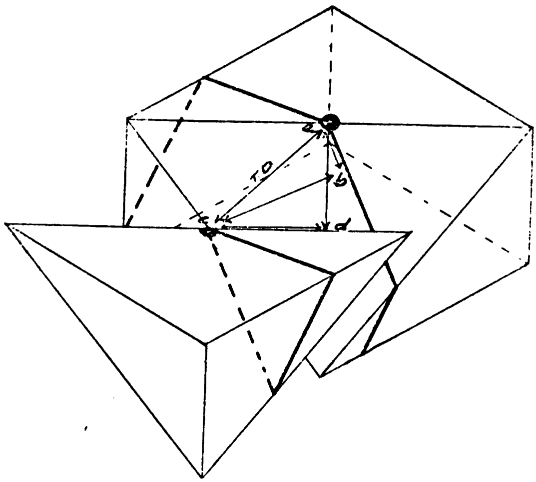


FIG. 1.

THE DISPLACEMENTS

ac = total displacement.
cb = perpendicular displacement.
ab = parallel displacement.
ad = normal displacement.
cd = horizontal displacement.

The throws. Fig. 3. These are measurements in any vertical section, and are of importance because in making structural sections the throw in that section must be deter-

mined. With reference to any particular fault plane under consideration, the section normal to the strike of the fault is the most important, and if the direction of the section is not designated in describing a throw, this section is understood. Taking the left front bounding plane of Fig.

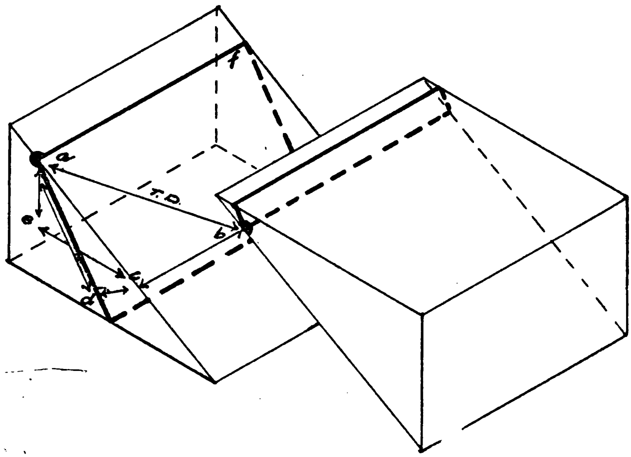


FIG. 2.

THE SEPARATIONS

ac = total separation.
cd = perpendicular separation.
ad = parallel separation.
ae = normal separation.
ec = horizontal separation.

3, *ac* is 'total throw,' *bc* is 'parallel throw,' and *ab* is 'perpendicular throw,' *cd* is 'horizontal throw,' and *ad* 'normal throw.'

The heaves. Fig. 4. The heaves are measurements in a horizontal plane. In the figure *ac* is 'total heave', *ab* is 'parallel heave', and *bc* is 'perpendicular heave'.

III. METHODS OF PROJECTION

In fault problems we have to deal with at least two planes, namely, the fault plane and the faulted vein, dike, or stratum. (Hereafter the word 'stratum' will be used for any faulted body having the form of a sheet, dike, bed, or vein.) This involves representation in three dimensions, and only the four methods most valuable for our purposes will be mentioned, two of which are discussed and used.

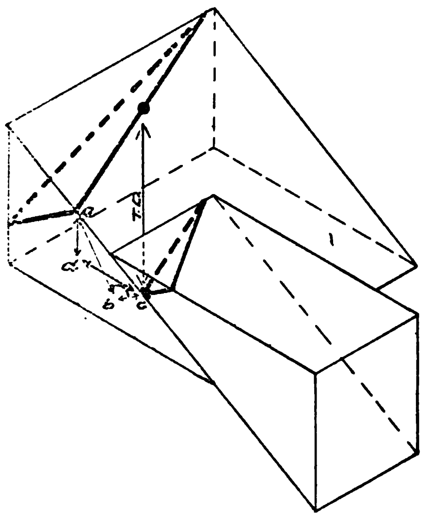


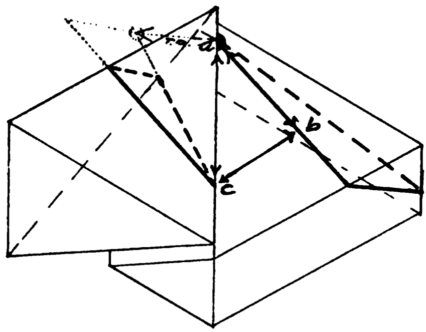
FIG. 3.

THE THROWS

ac = total throw.
ab = perpendicular throw.
bc = parallel throw.
ad = normal throw.
dc = horizontal throw.

1. *Mine Models.*—The use of mine models has become

increasingly popular in the last twenty years; their development being due largely to the need of giving a clear idea of workings, veins, and faults, to non-technical judges and juries. More recently they have been found to be valuable in impressing still less technical stockholders. To those with experience underground, and who have a trained geometrical sense, models have little if any advantage over mine maps.



[6]Fig. 4.

The Heaves

ac = total heave.
bc = perpendicular heave.
ab = parallel heave.

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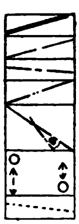
[6]It must not be inferred from Fig. 1 to 4 that a single spot can be identified in both parts of a faulted stratum and the total displacement measured directly. In the majority of cases the complete solution determining the position of the lost portion of a faulted vein or lode involves the discovery of the faulted and unfaulted positions of two other strata (dikes, sills, beds, veins, or zones of any kind), or the faulted and unfaulted position of one stratum of any kind and the direction of movement on the fault plane. The latter is often shown by striae. This data is often gathered by means of a geological examination along the trend of the fault plane, possibly far away from the lode and workings.

2. *Stereogrammatic Projection.*—An effective method of indicating spacial relations is by means of a diagram of a block; mine workings, fault planes, veins, etc., are located by measurements along the edges of the block. Of the various methods of projecting these blocks, the isometric is the most valuable, and is the only one used in this paper.

3. *Orthographic Projection on Two Mutually Perpendicular Planes.*—This is a simple and at the same time a valuable method for representing underground conditions. However, the projecting of the two perpendicular planes on the single plane represented by the sheet of paper is unsatisfactory where the workings are extensive, and especially where they are not confined to a definite plane, and for most purposes, especially solving problems, 4 is better.

4. *The Two-Level or Contour Method.*—Mines are mapped by 'levels'. Exploration and extraction workings are generally driven at regular intervals (100 ft. is the most common interval) and the group of workings at each depth is a 'level'. Given the trace of a vein on two levels and it is completely located unless it suffers some distortion lower down, in which case the new position is shown by the traces on the lower levels. By using the simple method of mapping faults and faulted strata on two levels (one 100 ft. below the main or reference level), the graphic solution of fault problems is made easy, and is the one adopted in the following discussion. The legend is as follows:

Visible trace of plane with side of block.
Invisible trace of plane with side of block.
Intersection of plane within block.



Trace of plane with reference surface (strike line).
Trace of plane with surface 100 ft. below.
Projection of intersection of any planes.
Projection rotated back into reference surface.
Projection of a line—portion above reference surface is light solid line; that below is dot and dash. Dip of line theta is given by rotating same around its projection. (See Fig. 5).
A point is located by a small circle and the distance above or below reference plane by a dotted line.
Construction lines.

Isometric Projection.—Space is represented in Fig. 5 by a cubical or rectangular block, position in which is determined (1) by locating the point in a horizontal plane, and (2) locating this plane according to its distance below the top of the block. The edges of the block are projected

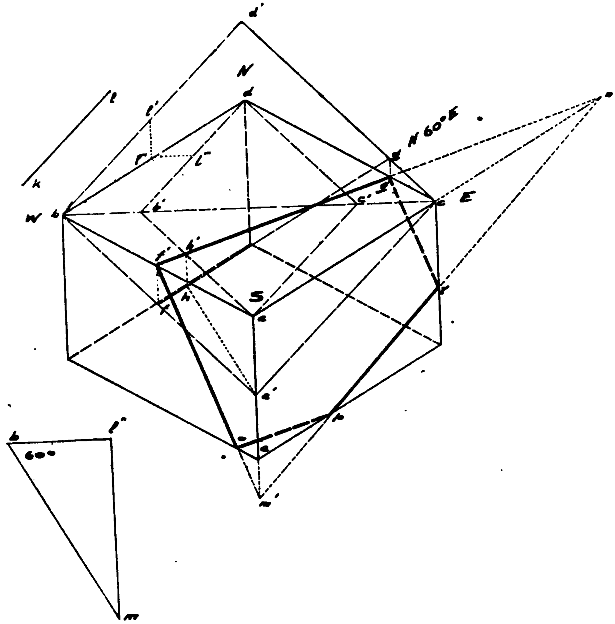


Fig. 5.

orthographically on the plane of the paper, by directing the line of sight at a corner of the cube. The edge of the block representing depth is projected vertically, and the other two edges are projected upward to the right and left, the angle between the projections being 120 degrees.

The projections of the edges of the block are shorter than the projecting edges, and all measurements parallel

to these edges are measured in the 'isometric scale'. Those lines parallel to the diagonal *bc* are the only ones projected full length. Of all directions in the horizontal plane, the 'short diagonal' *ad* is most inclined toward the line of sight and is therefore most reduced in projection. Points are usually located by taking the coördinate measurements parallel to the edges, but occasionally it is convenient to take measurements parallel to either of the two diagonals, and it is therefore valuable to know the ratio of these three scales. To determine this ratio, revolve the block around the diameter *bc* until the top coincides with the plane of the paper, taking the position *a'bd'c*. The ratio of the lengths of the projections of a given distance, taken (1) parallel to the 'long diameter' *bc*, (2) the edges *bd*, and (3) the 'short diameter' *ad* is *bd'* : *bd* : *b'd*.

To find the length of the projection of the line kl in these three directions. Take *bl'* equal to *kl*. Draw *l'l''* perpendicular to *bc*. *bl''* is the length of the projection parallel to the edges. Take *l''l'''* parallel to *bc*, *b'l'''* is the length of the projection when *kl* is parallel to *ad*.

To draw a plane whose dip and strike are known. Locate the points of the compass on the block. (In Fig. 5, north is parallel to the 'short diagonal'.) Lay off *fg* in the revolved position (*a'bd'c*) of the top of the block according to its strike (north 60° east). Draw *a'h* perpendicular to *fg*. Revolve *a'bd'c* about the line *bc* back into its original position *abcd*. *f*, *h*, *g*, and *a'* take the positions *f'*, *h'*, *g'*, and *a*. The dip of the stratum *f' g'* is given as 60° to the southeast, and since *ah'* is perpendicular to *f' g'*, a plane passed through *ah'* and *ae* will show the dip. The length *kl* was taken equal to *a'h*, therefore *bl''* is its isometric equivalent (see paragraph above).

Constructing a triangle with a 60° angle, and the isometric equivalent of *a'h* (*bl''* in figure) being used as the adjacent side, the opposite *ml''* shows the proper isometric length to be laid off on the edge of the cube. Point *m'* is taken this distance below *a*. By connecting *m'* and *f'* trace *f'o* is obtained. By projecting the edge *ac* and the

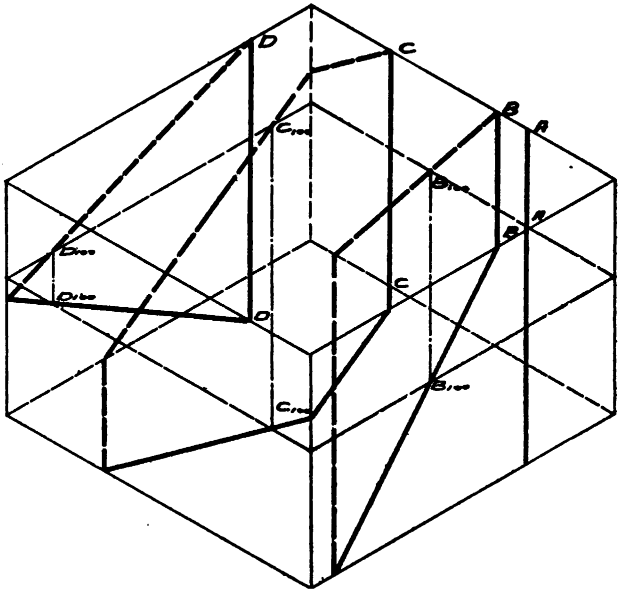


Fig. 6.

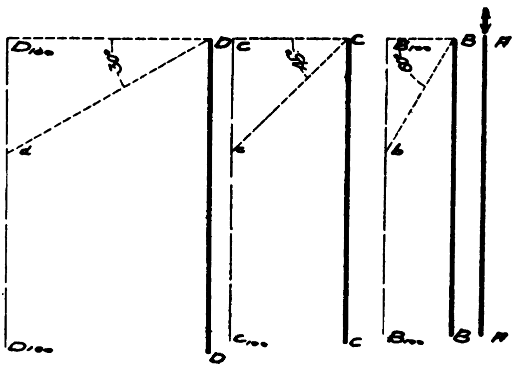


Fig. 7.

line $f'g'$ until they meet in point n and connecting m' and n, trace pr is found, and other traces are then drawn in.

The two-level or contour method of projection. The relation of the isometric to the two-level method of projection is shown by Fig. 6 and 7. In the latter, every plane, unless perpendicular, is located by two lines. The heavy line (*BB*) is the map of the plane on the reference level, and may be called 'the strike line.' A line of light dashes marks the trace of the plane on the level below and this is the contour line of the plane. Any number of contours may be used if the plane changes dip or strike with depth. The dip of the stratum is shown by the distance between the strike line and the contour, and the 'dip triangle' may be constructed by drawing a line perpendicular to the two traces ($B\,B_{100}$) and laying off to scale 100 ft. on the contour ($B_{100}b$) and completing the triangle ($B_{100}\,B\,b$).

IV. ELEMENTARY PROBLEMS IN LINE AND PLANE INTERSECTIONS

Elementary problems in line and plane intersections to show the application of the two-level method of projection. (Fig. 8 to 11.)

Problem 1. Fig. 8.—Given two parallel planes cut by a third plane. *To find* the shortest distance (both magnitude and bearing) between the two planes measured on the third plane.

Analysis.—Revolve the intersections of the parallel planes with the third plane into the horizontal position, using the strike line of the third plane *CC* as the axis of revolution. Draw a perpendicular between these traces and revolve back into the original position.

Construction.—*AA* and *BB* are the strike lines of the two parallel planes whose dips (45° in this case) are shown by the contours $A_{100}A_{100}$ and $B_{100}B_{100}$. *CC* and $C_{100}C_{100}$ represent the strike and dip of the third plane.

The strike lines *AA* and *CC* intersect at p_1 and the contours at t_1. p_1t_1 is the projection of the intersection of these planes, and p_2t_2 that of the second plane *BB* with

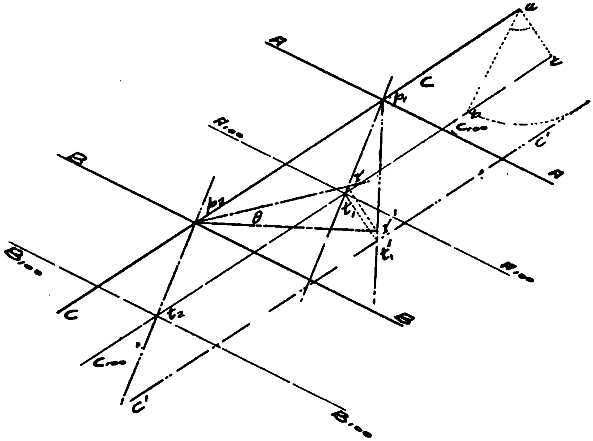


FIG. 8.

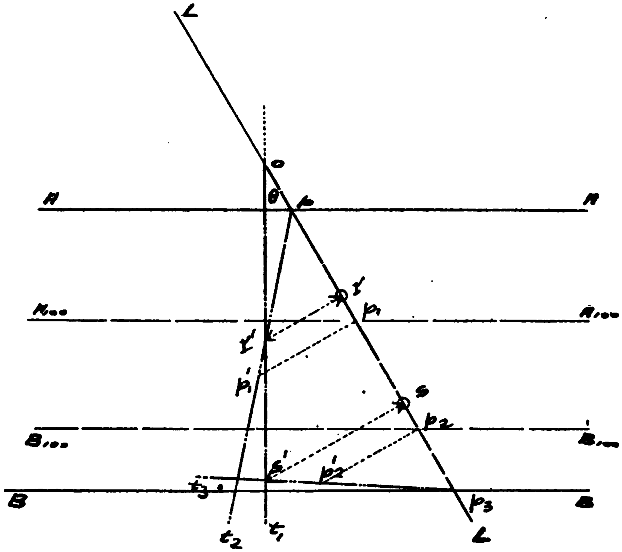


FIG. 9.

plane CC. Construct the dip triangle abc. ab measures the distance $C_{100}C_{100}$ from CC when the plane is revolved about its strike line into the surface. Such a revolution will cause p_1t_1 to take the position p_1t_1'. Draw p_2r' perpendicular to p_1t_1', and then revolve CC back into its original position, when r' falls on r.

p_2r' gives the length of the desired measurement.

p_2r is its projection and gives its bearing.

Problem 2. Fig. 9.—Given two planes and an intersecting line. *To find* the points where the line pierces the planes.

Analysis.—Take an auxiliary vertical plane containing the given line and revolve it into the horizontal position. The intersections of the traces of the given planes on the auxiliary plane, with the line, locate the desired points.

Construction.—LoL is the given line piercing the surface at o with a dip of Θ. AA and BB are the strike lines of the given planes with their dips shown by the contour lines. Pass tne auxiliary perpendicular plane through LoL and revolve it into the horizontal position about LL. ot_1 is the revolved position of that portion of the line LoL that lies below the surface of the reference plane. The intersections of the given planes with the auxiliary plane are found by dropping perpendiculars (scaling 1000 ft. in length) from p_1 and p_2 (points situated at the intersection of the contours with oL, and therefore projections of points 100 ft. below the surface), and by connecting the ends of these perpendiculars (p_1' and p_2') respectively with p and p_3. Therefore pt_2 and p_3t_3 are the desired traces, and r and s are the projected positions of the points where LoL pierces planes AA and BB at depths respectively of rr' and ss'.

Problem 3. Fig. 10.—Given three points. *To find* the plane containing them.

Analysis.—Draw two perpendicular construction planes, each containing a different pair of points, and rotate the planes to the surface. Draw a line passing through the pair of points in each construction plane. The points where these lines pierce the surface determine the strike-

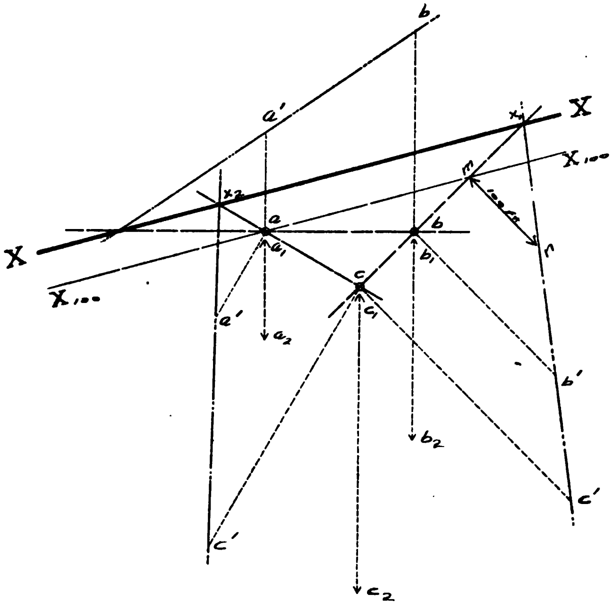

b
a'
X
X_{100}
x_2
a
b
a_1
b_1
c
c_1
a'
a_2
b'
b_2
c'
c'
c_2
X
X_{100}
c

FIG. 10.

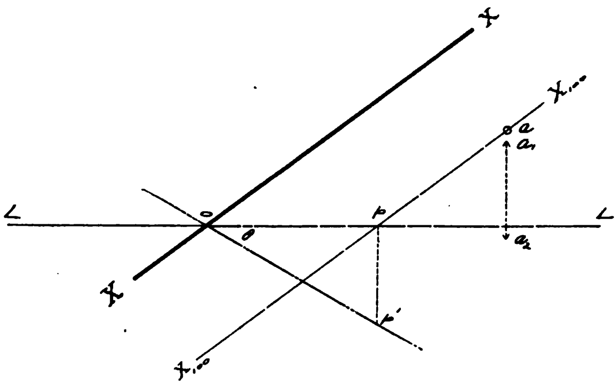

X
X_{100}
a
a_1
L
o
p
L
a_2
X
p'
X_{100}

FIG. 11.

line of the plane, and its dip is shown by passing the contour parallel to the strike through the projection of a point 100 ft. below the surface on either one of the lines.

Construction.—*a, b,* and *c* are the given points whose distance below the surface are, respectively, a_1a_2, b_1b_2, and c_1c_2. Pass perpendicular planes through *ac* and *bc* and revolve them into the horizontal position about their respective strike lines *ac* and *bc*. x_2c' and x_1c' are the revolved positions of the lines in each plane containing their respective pair of points ($aa' = a_1a_2$, $cc' = c_1c_2$, etc.). The surface trace of the desired plane passes through x_1 and x_2, and its contour is located by point *m*, which is the projection of a point on the line passing through *b* and *c,* 100 ft. below the surface.

Problem 4. Fig. 11.—*Given* a line and a point. *To find* the plane containing them.

Analysis.—The strike-line of the desired plane must pass through the point where the given line pierces the surface, and be parallel to a line drawn through the projection of the given point, and through the projection of a point of equal depth situated on the given line. The plane contour passes through the projection of a point on the line 100 ft. below the surface.

Construction.—*LoL* is the given line with a dip of Θ, and *a* the given point at a depth a_1a_2 below the surface. *p* is the projection of the point *p'* on the line *LoL* and taken at a depth of a_1a_2, and as this happens to be just 100 ft., the contour of the desired plane passes through these two points, and the strike-line is *XX* parallel to the contour and passing through *o.*

V. FAULT MOVEMENT OF PURE TRANSLATION; FAULT PLANE VERTICAL

Fault Problems.—Fault movement that of pure translation; fault plane vertical. (Problems 5, 6, and 7.)

Problem 5. Fig. 12.—*Given* a vertical fault plane and the strike, dip, and total heave of two faulted strata. *To find* the amount and direction of total displacement.

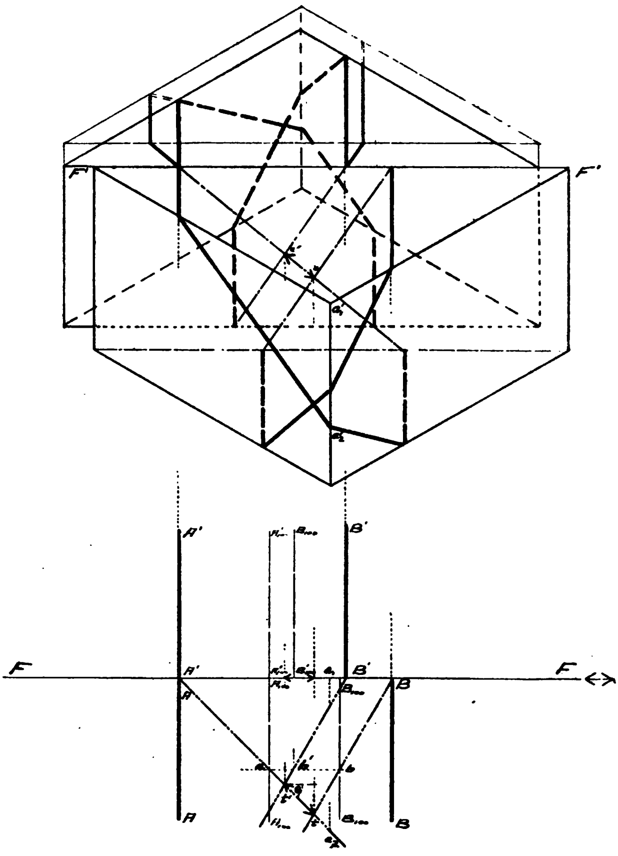


FIG. 12.

Analysis.—The fault movement separates the traces, on the fault plane, of the two strata on one side of the fault, from the traces of those on the other side; therefore a line connecting the intersections of the traces determines the amount and direction of the total displacement.

Construction.—The fault plane FF is vertical and AA and $A'A'$ are the two portions of the faulted stratum whose total heave $=$ *nil,* and BB and $B'B'$ of the second stratum whose total heave $= BB'$. Revolve the fault plane FF about its strike-line into the horizontal position. ab is drawn at a distance of 100 ft. from FF, and therefore is the revolved position of the 100-ft. contour of the vertical fault plane. a, b', and b are at the intersections of ab with the contours of the faulted strata. At and Bt are the revolved positions of the traces of AA and BB on the fault plane and intersect at t. Likewise the revolved traces of $A'A'$ and $B'B'$ intersect at t'. Before faulting, t adjoined t'. Therefore the total displacement $= tt'$ in amount and $\Theta =$ the angle it makes with the 'horizontal displacement.'

Stereogrammatic representation of this problem is shown above in Fig. 12. The graphic solution beneath can be projected on the isometric block in the following simple manner. Make the 'long diameter' of the block $F'F'$ parallel to FF. Then all points on FF (Fig. 12) can be projected directly on $F'F'$, for the long diameter is projected full length. To find, for instance, point a_2' on the stereogram in order to draw in the front trace of AA, measure its true distance a_1a_2 in the lower figure and diminish it according to the isometric ratio, giving the distance $a_1'a_2'$. In the upper figure the elevated block is supposed to be cut down to a level with the other block by erosion, but the removed upper portion with traces is shown in light lines.

Problem 6. Fig. 13.—*Given* a perpendicular fault plane, and the dip, strike, and total heave of two faulted strata, one whose strike is parallel and the other perpendicular to the fault. *To find* the total displacement.

Analysis.—Fault FF is a strike fault when considered in reference to the parallel stratum, and a dip fault in reference to the stratum with perpendicular strike to the fault.

By passing a vertical plane normal to the fault plane,

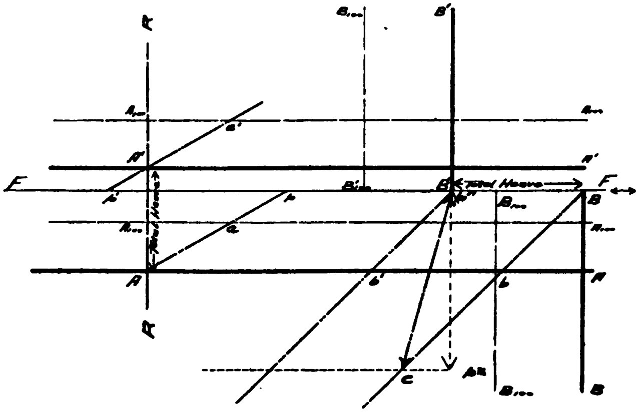


FIG. 13.

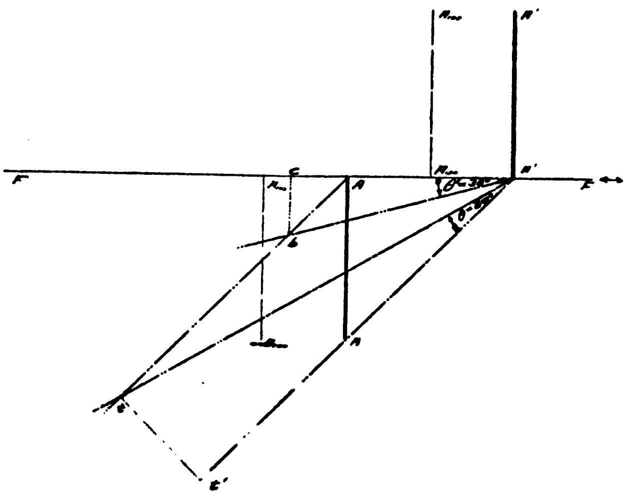


FIG. 14.

the normal displacement of the fault is found, and from this, the total displacement which separated the traces of the second stratum on the fault plane, is found.

Construction.—Pass a construction plane RR perpendicular to the fault strike line FF. Aa and $A'a'$ are the revolved traces of AA and $A'A'$ on RR. Continue the traces until they intersect the fault trace on RR at p and p'. Then pp' is the normal displacement. Revolve the fault plane on its strike-line FF into the horizontal position. The revolved traces of BB and $B'B'$ on the fault plane are Bb and $B'b'$. Lay off $p'' \ p''' = p' \ p$ (normal displacement.) The total displacement is $p''c$ and $cp''' =$ horizontal displacement.

Problem 7. Fig. 14.—Given a perpendicular fault plane, the dip and strike of both portions of a faulted stratum, the total heave, and the direction of fault movement *(a)* Θ is measured downward from the horizontal displacement, *(b)* Θ is measured upward from the parallel displacement. *To find* the total displacement.

Analysis.—The distance between the traces of the two portions of the faulted stratum on the fault plane in the known direction (often told by fault striæ) of fault movement, is wanted.

Construction.—Locate At and $A't'$ the revolved position of the traces of AA and $A'A'$ on the fault plane. Lay off Θ' (the angle between horizontal displacement and total displacement) $= 30°$ and find the total displacement $A'b$ and the normal displacement bc. Lay off Θ (the angle between parallel and total displacement) $= 30°$ and find $A't$ as the total displacement, and tt' as the perpendicular displacement.

VI. FAULT MOVEMENT OF PURE TRANSLATION; FAULT PLANE INCLINED

Fault Problems.—Where the fault movement is that of pure translation, and the fault plane is inclined. (Problems 8, 9, 10, 11, and 12.)

Problem 8. Fig. 15.—Given an inclined fault plane and

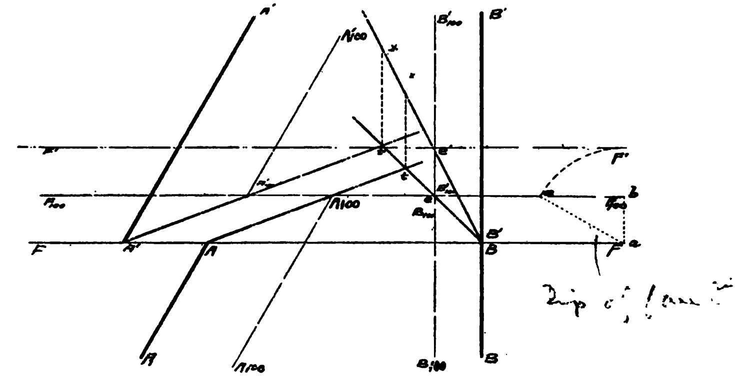

A'
A'100
B'100
B'
A100
F
A
B
F'
b
a
A
A100
B100
B

Fig. 15.

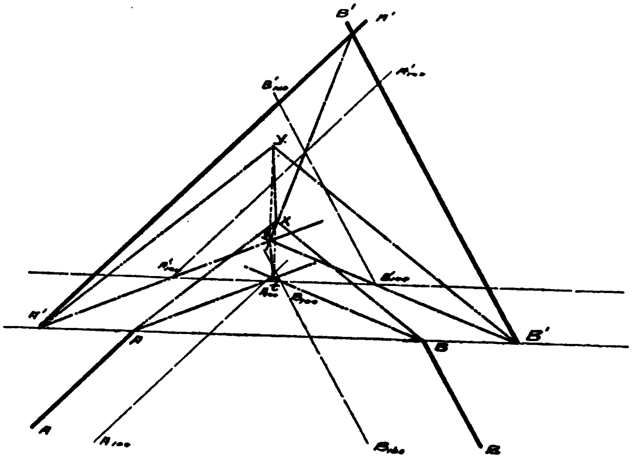

B'
A'
B'100
A'100
A'
A
B
B'
A
A100
B100
B

Fig. 16.

the dip, strike, and total heave of both portions of two faulted strata (the total heave of one of the strata = *nil*). *To find* the amount and direction of the total displacement. The analysis is the same as for problem 5.

Construction.—The projections of the intersections of the inclined fault plane with the faulted strata are *At, A′t′,* and *Bt*. *tt′* is the projection of the total displacement. To revolve the fault plane into the horizontal reference surface construct the dip triangle *abc*. The contour of the fault will move out until a distance from its surface trace = *ac,* assuming the position *F′F′*. *Be* moves out to *Be′*, and *tt′* to *xy,* which measures the magnitude of the total displacement.

Problem 9. Fig. 16.—Given an inclined fault plane and the dip, strike, and total heave of both portions of two faulted strata. *To find* the amount and direction of the total displacement. The analysis is the same as for problem 5.

Construction.—Find the intersections of the projections of the fault plane traces of the strata on each side of the fault plane. These intersections are *t* and *t′*, and therefore *tt′* is the projection of the total displacement, the length of which may be found by rotating it into position *xy* which it assumes in the horizontal plane.

Problem 10. Fig. 17.—Given an inclined fault plane, and the dip and strike-line of one side of a faulted stratum, as well as the bearing and amount of the total displacement. *To find* the outcrop of the portion of the stratum on the other side of the fault plane.

Analysis.—After plotting the direction of total displacement, and revolving its original (in the fault plane) into the horizontal position, the total displacement may be measured on this line from the intersection of the stratum and the fault plane, and the point thus found, when revolved back into the fault plane, will lie on the trace of the unknown portion of the stratum on the fault plane. Or, by determining the depth this point is below the surface, and projecting a line to the surface, in the direction of dip and at the angle

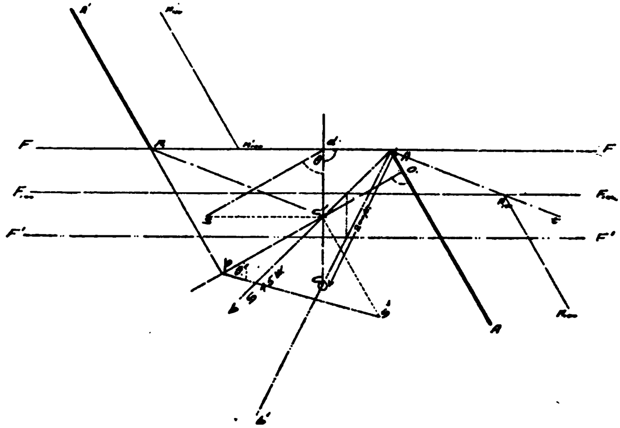


FIG. 17.

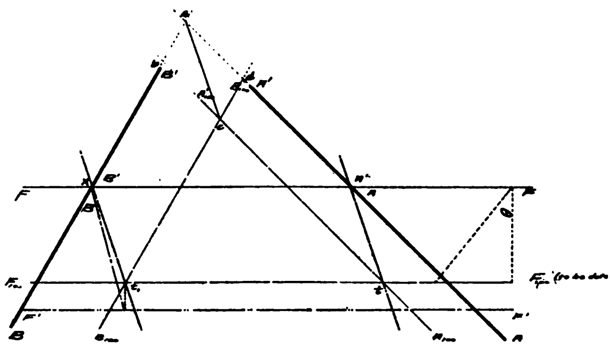


FIG. 18.

of dip of the stratum, a point in the extended outcrop of the second portion of the stratum is determined.

Construction.—*Ab* is drawn in the known direction of total displacement, and its original is revolved into the horizontal position *(Ab′)* and the amount of the total displace-

ment (200 ft.) measured on the same locates point *c,* which is revolved back to c', and the trace $c'A'$, parallel to At is drawn, its intersection with FF locating $A'A'$. Or the distance c' lies below the surface ($c's$ and $c's'$) is used to construct the triangle $c's'p$, drawn on op, which is perpendicular to AA, and Θ is the dip of AA. Point p lies on the produced strike of $A'A'$.

Problem 11. Fig. 18.—Given the surface trace of the fault plane, and the strike and dip of two faulted strata, neither of which is heaved. *To find* the dip of the fault plane.

Analysis.—Since neither stratum is heaved, (1) the traces (on the fault plane) of the two portions of either stratum coincide, and (2) they are parallel to the fault movement.

Construction.—Construct parallelogram *abcd, ac* is the one line common to both strata, and point c is 100 ft. below point *a.* Lay off xt_1 and parallel to *ac.* The contour of the fault plane passes through t_1 and its dip is Θ.

Problem 12. Fig. 19.—Given the strike of the fault plane, and the strike, dip, and total heave of three faulted strata. *To find* the dip of the fault plane.

Analysis.—The linear intersections of the strata on one side of the fault plane taken in pairs, will intersect in a point. A line drawn to the corresponding point on the other side of the fault plane will be parallel to and equal in length to the total displacement. The dip of the fault plane is determined by using the total displacement thus determined in combination with the dip, strike, and total heave of any faulted stratum.

Construction.—Find any two of the three mutual intersections of the strata AA, BB, CC (at, bt, ct). These intersect in a common point (t), for each pair of lines lies in a common plane. t' is the corresponding point of $A'A'$, $B'B'$, $C'C'$. tt' is the projection of a line parallel and equal to the total displacement. Take ss' parallel and equal to tt'. Bs is the projection of the trace of BB on the fault plane, therefore its intersection with $B_{100}B_{100}$ at x locates

$F_{100}F_{100}$, and the desired dip (Θ) is found by constructing the dip triangle.

Discussion of the problems in which fault movement is that of pure translation. The above problems have been selected, not only to illustrate graphic methods, but to bring out certain features in faulting, which if borne in mind, may assist in approximate solutions, and emphasize those phenomena for which the investigator should be on the watch. It will be seen that there is no difference in attacking problems whether the fault plane is vertical or inclined, and that when the movement is of pure translation, the

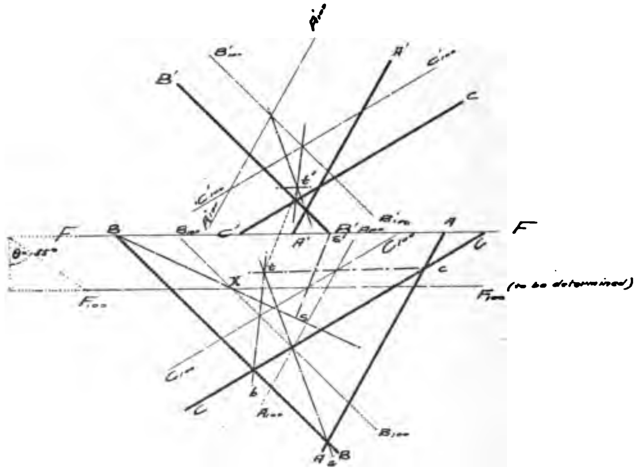


FIG. 19.

traces of each portion of a faulted stratum, in any plane, are parallel, and therefore a difference in strike on two sides of a fault indicates a pivotal component of fault movement.

Problems 5, 8, and 9 are examples of the solution of the important problem, which calls for the determination of total displacement when the fault plane is known, as well as

the dip, strike, and total heave of two faulted bodies. The cases investigated where one of the strata is not heaved, are taken to emphasize that the offset at the surface is a function of the angle between the total displacement and the trace of the faulted stratum on the fault plane, and if the two are parallel, there will be no heave, regardless of the amount of fault movement. Problem 6 is an unimportant variation of problem 5, and problems 7 and 10 involve simple constructions often used in graphic solutions. Problems 11 and 12 involve the determination of the dip of the fault plane. In practice, when direct measurement of dip is not satisfactory, both dip and strike are generally determined by locating accurately three points on the outcrop of the fault plane, and determining their elevation and plotting these data according to methods of descriptive geometry. Problem 11 is theoretical, and is presented to show the possibility of the occurrence of fault movement, which nevertheless may not heave bodies with entirely different strikes (provided their intersections on the fault plane are parallel and in the direction of the total displacement). Solution 12 may be occasionally of considerable importance.

VII. FAULT PROBLEMS WHERE A ROTATION ABOUT A POLE IS COMBINED WITH TRANSLATORY MOVEMENT

Fault problems where a rotation about a pole is combined with translatory movement.

When rotary movement (about a pole perpendicular to the fault plane) is combined with that of pure translation, the angle between the fault plane traces of each portion of the faulted stratum measures the angular displacement regardless of the position of the pole. Almost every careful underground fault measurement will disclose such a variation of the traces from the parallel position, but generally so slight as to cause uncertainty as to whether this difference should be ascribed to rotation, or to minor irregularities of movement, or to faulty measurements. As

already stated, cases have been reported of important pivotal motion, and as accurate, detailed observations increase, rotation will probably be found to be of more importance than suspected heretofore.

Solutions of problems in combined rotation and translation are greatly simplified by the following facts regarding faults. (1) Movements are so frequently confined to definite planes that an ideal geometrical plane can generally be selected to represent the fault plane. (2) Fault fissures are universally closed. Any given earth movement will find complete relief (*a*) by a moderate amount of form distortion *before* the break takes place, and *after* the break (*b*) a movement between the two sides adjoining the fault fissure, and (*c*) a rock flowage by which that portion of the stress is relieved, that could not be satisfied by the fault displacement. This rock flowage caused by (*a*) and (*c*) must be considered in an investigation of the total deformation of an area, but in the practical investigation of the amount and direction of the total displacement of a lode along a certain fault plane, the consideration of rotary and translatory movements is made regardless of rock flowage. Further rock flowage either before or after the break causes a deformation of the shape of the stratum which generally does not interfere with the following of the same underground as does a break in the stratum.

From the above it follows that the pole about which rotation takes place is perpendicular to the fault plane. Otherwise one side of the fault plane must grind and the other gap. The initial position of the pole, which it would assume to give complete relief to the stresses, is probably not often perpendicular to the line of weakness that determines the fault plane, but molecular adjustment, which prevents the fissure from opening, etc., has the effect of raising the pole to the perpendicular position.

Certain 'strip faults',[7] by means of which an entire desert mountain range is given monoclinal dip, approach the form of a right cylinder. Such a form, if perfectly developed,

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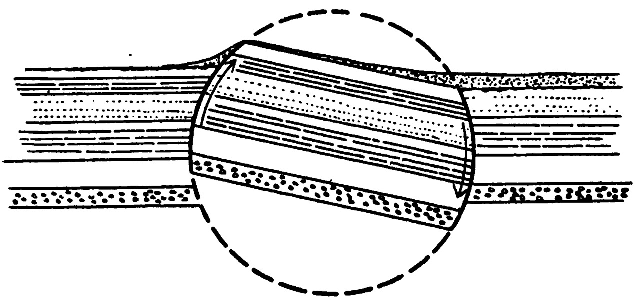
[7]Tolman, C. F. *Jour. of Geol.*, Vol. 17, No. 2, p. 139.

could relieve both a rotation perpendicular to the end faces, and a movement parallel to the curved face, without an accompanying rock flowage. (See accompanying figure.) In the occurrences in mind, the cylindrical surface is represented by one or more great parallel faults, occurring as the main structural features of the region. The cross faults, on which the rotary movement should be expressed, are seldom developed, the rotation being satisfied by flowage, instead of fracture, and even when fracture occurs the cross-faults are much more difficult of recognition than the parallel faults.

Fault block systems, however, are more common than



CYLINDRICAL FAULT, AS DEVELOPED IN SOUTHERN ARIZONA.

single large strip faults, the blocks presenting a large variety of form, and here evidently only a portion of the movement can find relief along the fracture planes, the rest going to adjust the form of the block to its new position.

Problems 13 and 14.—Given a fault plane, a pole,[8] and angular displacement of ρ about the pole, and the dip and strike of the two strata on one side of the fault plane. *To*

[8]It is to be noted that the pole is not a point, as is sometimes the case in mathematical discussions, but is a line passing through the rotated portion of the block, extending to and perpendicular to the fault plane.

find the outcrop and dip of the same on the other side of the fault plane, after both sides have been planed down to a common horizontal surface.

Analysis.—An auxiliary plane parallel to the fault plane should be constructed. The points where the pole pierces both planes can then be located, and these used as centres about which the fault plane and auxiliary plane traces of

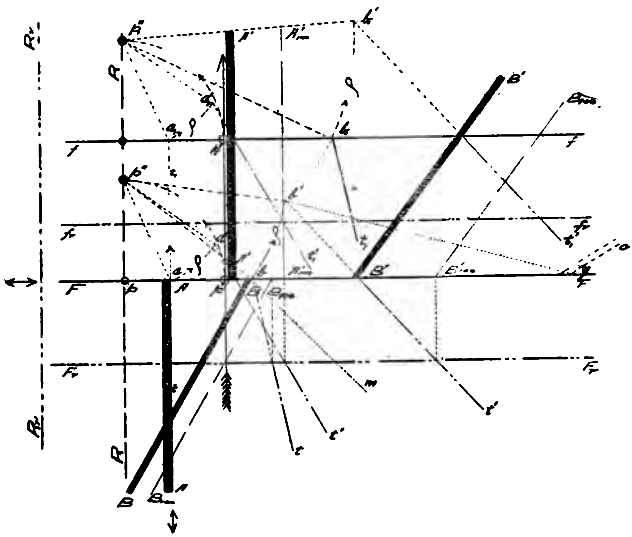


FIG. 20.

the given strata can be rotated. The points where these rotated traces of each stratum intersect the common level to which both sides are reduced, locate the strike-line of the rotated portion of that stratum, the dip being determined from the rotated trace on the fault plane.

Case I.—The fault plane is perpendicular. Fig. 20.

Construction.—FF is the strike-line of the perpendicular fault plane and ff of the auxiliary plane F_rF_r and

$f_r f_r$ being respectively the revolved position of their 100-ft. contours. Pass a perpendicular plane RR through the pole, and revolve it about its strike-line. $R_r R_r$ is the revolved position of its 100-ft. contour, and the arrow shows the known revolved position of the pole, intersecting the fault plane and the auxiliary plane respectively at p' *and* p_1'. The fault plane and the auxiliary plane, however, are revolved about their strike lines into the horizontal position, such a rotation bringing p' into the position p'' and p_1' into the position p_1''. The traces at and bt are now rotated about p'' through angle ρ into the positions $a't'$ and $b't'$, and likewise in the auxiliary plane. a_1t_1 and b_1t_1 are rotated about p_1'' into the positions $a_1't_1'$ and $b_1't_1'$. Let erosion bring down the rotated block to the level represented by lines FF and ff. The intersection therefore of $a't'$ on FF and $a_1't_1'$ on ff locates the new strike-line $A'A'$, and $B'B'$ passes through the intersection of $b't'$ with FF and $b_1't_1'$ with ff. Their dips are determined from their traces $A't'$ and $B't'$.

Case II.—The fault plane is inclined. Fig. 21.

Construction.—Inasmuch as the fault plane is now inclined, the pole shown in the revolved position of the auxiliary plane RR, has the inclined position $p'p_1'$, instead of a horizontal position as in the preceding case. It will be easily seen that when the fault plane and auxiliary plane are revolved on their strike-lines into the horizontal positions, the centres of rotation in these planes will be respectively p'' *and* p_1''. The rotation of the revolved traces at and bt about these centres and the determination of the strike and dip of the rotated portion of each is carried on as described for case I.

Discussion.—These two simple problems emphasize the following facts of practical bearing, which may assist in the detection of the existence of rotary movement, and explain some of the phenomena of faulting that otherwise would be difficult to understand. Assuming that fault movement is by angular displacement: (1) With a vertical fault plane, strata striking at right angles to the same

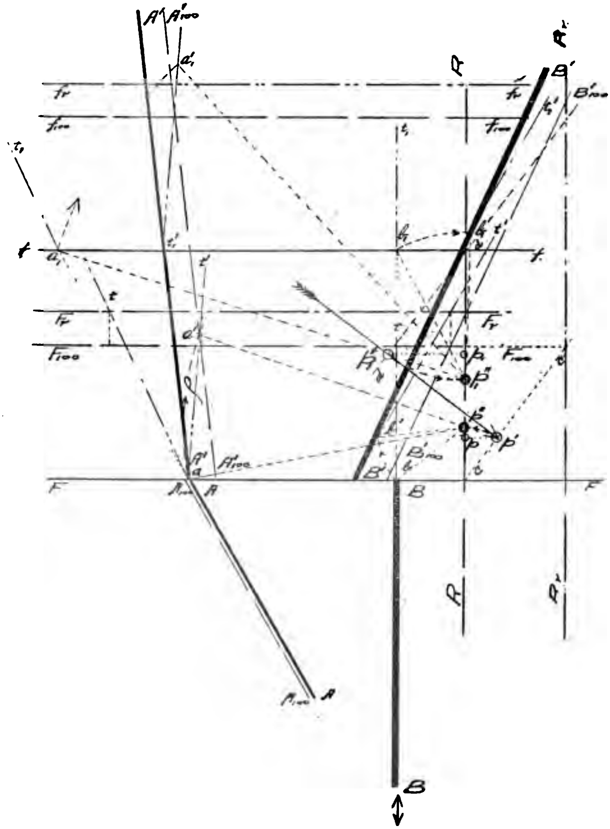


FIG. 21.

suffer change in dip and not in strike. Others develop changes in both dip and strike after rotation. (2) Only those lines parallel to the pole will remain parallel to their original position after rotation. A strike-line can be parallel to the pole only when it is perpendicular to a vertical fault plane. Therefore when a stratum suffers a change in

dip without a noticeable change in strike, the fault plane should be approximately vertical. (3) Very considerable heaves may be developed by a relatively small rotation, if the trace of the stratum is thereby caused to approach a horizontal position. For instance, in Fig. 20 if the stratum BB intersected the fault plane in the trace bm instead of bt, the faulted portion would have had outcrop no instead of BB'. Are not some of the thrusts described by geologists better explained by moderate rotations than by great translatory displacements?

Problem 14. Fig. 22.—Given a series of strata that have undergone a combined rotation and translation. *To find* 'the equivalent pole' of rotation.

Analysis.—If the position of both parts of two faulted strata are known, which have undergone a known angular rotation about a pole, the location of which is unknown, combined with a translatory displacement the amount and direction of which is also unknown, these factors cannot be determined; but it is possible to locate an equivalent pole about which the known rotation will produce the displacements observed.

All the poles about which the fault plane trace of one portion of a faulted stratum can be rotated through angle ρ so that it will fall on the projected fault plane trace of the second portion, will pierce the fault plane in a line. The intersection of two or more of the lines thus found, locate the equivalent pole.

Construction.—All the strata are rotated about pole PP through angle ρ and then suffer a 'total displacement' of pure translation indicated by arrows $a'a''$, $b'b''$, $c'c''$, etc. (In the figure these are shown in the revolved position of the fault plane). Stratum DD was faulted into the position D_fD_f before the above described movement took place.

RR is a perpendicular construction plane which is revolved about its strike line into the horizontal to show the position of the pole PP in relation to the fault plane $F'F'$ and the auxiliary plane $f'f'$. The points where this pole pierces the fault and auxiliary plane assume positions p

and p' respectively when these are revolved in the usual manner into the horizontal.

The fault and auxiliary planes being in the horizontal position, all rotations about p are made with the assistance of the construction circle *gihj,* and about p' the circle *mknl* is used. For example, *at,* the fault plane trace of AA is projected to g and h and rotated through ρ into the position *ij,* which locates $a't$, the fault plane trace of the rotated portion of the stratum (A_rA_r), and a' lies on the strike line A_rA_r. AA is projected until it cuts ff at a_1. a_1t_1 parallel to at is in the position the trace of AA on ff occupied before rotation took place. It is projected to k and l and rotated to position mn. The intersection of the latter with ff at a'_1 gives the second point on strike line A_rA_r. B_rB_r and C_rC_r are the rotated strike lines of BB and CC, and D_rD_r the rotated position of the faulted portion (D_fD_f) of DD. Each suffers a total displacement shown by the arrows, and after erosion levels both blocks to FF, assumes positions $A_{(\text{r and d})}A_{(\text{r and d})}$, $B_{(\text{r and d})}B_{(\text{r and d})}$, etc. After the final position is assumed, the angle ρ is known (lying between the two fault plane traces, at_2a''') and the total heave of each stratum (aa''' for example). If a pole pierces the fault plane at t_2, it can rotate at_2 through ρ so that it falls on $a'''t_2$ extended. ot_3 is a perpendicular erected at the centre of total heave aa''' bisecting angle at_3a''', which is made equal to ρ, and therefore will rotate at_2 through ρ so that it falls on $a'''t_2$ extended. These two points (t_2 and t_3) locate the line containing every point about which at_2 can be rotated through angle ρ so that it falls on $a'''t_2$ extended. (The easiest construction for obtaining this line is to draw arc ea''' with t_2 as a centre. It is easy to show that the line through ea''' is parellel to the line of poles through t_2.) While each pole will throw at_2 onto a different portion of $a'''t_2$ extended, erosion will cut the block to level FF and cause all to assume the same final position.

The lines of B and C poles are found in a similar manner and the three lines of poles intersect at p'', which is the *desired equivalent pole about which a rotation of* ρ *will*

produce the same result as the original combined rotation about p and translation of oa''. Since *DD* suffered an extra fault movement earlier than that affecting the other strata, its line of poles does not intersect at p''.

Discussion.—The uses that may be made of the 'line of poles' and the 'equivalent pole' may be summarized as follows:

(1) The line of polar intersection on the auxiliary plane may be used with advantage in finding the strike line of the faulted part of a stratum when the total heave and the angle ρ is known. It is evident that this line will be parallel to, and lie at a distance of pp' from the line of poles on the fault plane. To determine, for example, the strike line $B_{(\text{r and d})}B_{(\text{r and d})}$, locate b_1 at the projected intersection of BB on ff and draw b_1x parallel to bt'_2, and xb'_1 parallel to $b''t_2$. b_1' is the second point on the desired strike line.

(2) The equivalent pole may be used to determine the position of any stratum that has undergone the same set of movements that have affected the two strata from which its location was determined.

(3) Where there have been successive periods of faulting and lode formation, it may assist in grouping together the strata that have undergone the same set of movements.

(4) The lines of equivalent poles on the fault and any selected auxiliary plane may also be used with advantage in examining the distortions of the block during faulting. The consideration of these problems, however, lies beyond the scope of the present volume.

Finally, it should be remembered that purely mathematical studies of fault problems will be of little value in the solution of underground problems. All evidence should be collected and sifted by a careful geological examination before attempting to apply any mathematical treatment, and the very discrepancies between observed phenomena and those indicated by graphical investigations, may suggest the solution desired.